EFFECT OF NON-SYNCHRONOUS ROTATION
ON CLOSE BINARY STARS

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Abstract: The effect of non-synchronous rotation of the component star in close binaries on its limiting surface is analyzed and the [redacted] difficulties in the previous investigations are examined. It has been shown that no potential function exists in a coordinate system following the axial (non-synchronous) rotation of the component star. Consequently, no equilibrium surfaces can be obtained in this way. It is therefore suggested that we should retain the conventional coordinate system that rotates with the binary motion. Since a simple energy integral exists in this coordinate system, we can obtain a physical picture for the problem of the non-synchronously rotating star in the binary systems. Indeed we have found that the non-synchronously rotating star will modify its rotation gradually and become eventually synchronous with its orbital revolution.

In this multifarious activities and achievements that make him one of the greatest astronomers in the modern time, Professor Otto Struve has never forsaken the binary stars. After having spent some years of successful researches in other fields of astronomy now and then he always returned to this field of his early interest. Therefore, few astronomers have observed spectroscopically as many binary stars as he has done. Fewer still have left such a distinct mark as his in the history of double stars.

In a close association and collaboration with him for a decade during which I am proud to have received his confidence and learned the process of his mental perceptions and responses as a result of our daily afternoon meetings in cafes outside the North Gate of the Berkeley Campus of the University of California, I have always been

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impressed by his open mindedness to new ideas, his enthusiasm for and devotions to astronomy. Because of his open mindedness he saw any problem in its multi-faceted angles, thus making him not only an outstanding scientist but also a great leader. Because of his enthusiasm, he enlisted many astronomers into the fields of his interest. Our daily contact also induced me to become a novice in the study of binary stars in the mid fifties: It is therefore befitting for me ten years later to write this article on binary stars in this memorial volume as my attribute to a great leader as well as a dear friend whom I have both respected and admired.

I. A Critical Review of the Previous Investigations

The shape of stellar surfaces in the close binary system has been given as a first approximation by what has been predicted by the Roche Model (e.g. Struve and Huang 1957). According to this model, the axial rotation of component stars and their orbital revolution are synchronized. While the majority of close binaries indeed obey the rule of synchronization (Swings 1936, Struve 1950, Plaut 1959) there are some exceptions to which belongs β Lyrae - a peculiar binary system that Professor Struve (1941, 1958) had a life-long interest. Therefore, in recent years attempts have been made to predict the stellar surfaces, especially the limiting surface, when the component star does not rotate in synchronization with its orbital revolution (Kopal 1956, Plavec 1958, Kruszewski 1963, and Limber 1963).

In the restricted three-body problem one can define a potential function U in a coordinate system that rotates with the binary's orbital motion (e.g., Moulton 1914). Also an energy integral can be obtained in the form that the sum of the potential energy and the kinetic energy is constant. It is the existence of the energy integral in this form that enables us to predict the stellar surfaces of close binaries because only then the density and the pressure should be constant on any equi-potential surface. Thus, we should emphasize that the mere existence of a potential function does not necessarily warrant a prediction of the equilibrium surface of a star. Indeed, this is the reason that the calculations by previous investigators have to be regarded as unsatisfactory.

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Kopal (1956) first studied the limiting surface of the non-synchronously rotating star, using a potential function which Kruszewski (1963) has since pointed out to be incorrect. Later Plavec (1958), Kruszewski (1963) and Limber (1963) have all derived a potential function and obtained the limiting surface. However, even though their potential function is correct, their result is still open to debate because they have used for computing the limiting surface in one coordinate system a potential function expressed in another coordinate system which has a relative motion with respect to the first one.

Let us first choose a dimensionless system of units of measurement with the total mass of the binary as the unit of mass, with the separation between the two components as the unit of length and with $P/2\pi$ as the unit of time, where P stands for the orbital period. Thus, if we denote $1-\mu$ as the mass of one component, μ will be the mass of the other. Let us further choose a rotating (x,y,z) system such that the origin is at the center of the $1-\mu$ component, the x-axis points always towards the μ component, and the xy plane coincides with the orbital plane. We have the equations of motion for a test particle in the (x,y,z) system:

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} = \frac{\partial U_1}{\partial x}, \quad \frac{d^2y}{dt^2} + 2\frac{dx}{dt} = \frac{\partial U_1}{\partial y}, \quad \frac{d^2z}{dt^2} = \frac{\partial U_1}{\partial z}, \tag{1}$$

where

$$U_1(x,y,z) \equiv \frac{1}{2}\left[(x-\mu)^2 + y^2\right] + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{2}$$

and r_1 and r_2 are respectively the distances of the test particles from the two component stars. Note that U_1 is a function of time only through the coordinates (x,y,z) of the test particle.

We not introduce a new rotating (ξ, η, ζ) system such that it is rigidly fixed to the $1-\mu$ component, sharing the latter's axial rotation. Although the axial rotation of the $1-\mu$ component is not synchronous with the orbital motion, its rotating axis (chosen as the ζ-axis) is still assumed to be perpendicular to the orbital plane. Thus, if the (ξ, η, ζ) system is rotating with an angular velocity, ω, with respect to the (x,y,z) system, it rotates with an angular velocity, $1+\omega$, with respect to the rest frame of reference. The transformation equations between (x,y,z) and (ξ, η, ζ) are as follows:

$$x = \xi \cos \omega t - \eta \sin \omega t, \quad y = \xi \sin \omega t + \eta \cos \omega t, \quad z = \zeta \tag{3}$$

The equations of motion in the (ξ, η, ζ) system are

$$\frac{d^2\xi}{dt^2} - 2(\omega+1)\frac{d\eta}{dt} = -\mu \cos \omega t + \frac{\partial U_2}{\partial \xi} \tag{4}$$

$$\frac{d^2\eta}{dt^2} + 2(\omega+1)\frac{d\xi}{dt} = \mu \sin \omega t + \frac{\partial U_2}{\partial \eta} \tag{5}$$

$$\frac{d^2\zeta}{dt^2} = \frac{\partial U_2}{\partial \zeta} \tag{6}$$

where

$$U_2(\xi, \eta, \zeta) \equiv \frac{1}{2}(\omega+1)^2(\xi^2+\eta^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{7}$$

may be regarded as the negative value of the potential function. Actually, it is a misleading name as we shall see presently.

$U_2(\xi, \eta, \zeta)$ as given by equation (7) has been used by Plavec to evaluate the limiting surface of the non-synchronous $1-\mu$ component. We can now see that such an evaluation may not be regarded as satisfactory. In the first place, we encounter the time-dependent terms $\mu \cos \omega t$ and $\mu \sin \omega t$ in equations (4) and (5).

Secondly r_2, which is given by

$$r_2^2 = (\xi - \cos\omega t)^2 + (\eta + \sin\omega t)^2 + \zeta^2, \tag{8}$$

is now an explicit function of time. Therefore $U_2(\xi, \eta, \zeta;)$, unlike U_1 in the (x,y,z) system, is no longer an implicit function of time through the space coordinates alone but involves time explicitly. For brevity we write $U_2(t)$ for $U_2(\xi,\eta,\zeta)$. Because of the two complications, we can no longer derive from equations (4)-(6) the simple result that the sum of the potential energy and the kinetic energy is a constant of motion. Consequently, the surfaces defined by equation (7) do not represent equilibrium surfaces of the star. We can see this point analytically in the following way.

Multiplying equations (4)-(6) respectively by $d\xi/dt$, $d\eta/dt$ and $d\zeta/dt$ and adding the resulting equations together, we obtain after integration with respect to time that

$$\begin{aligned} &\frac{1}{2}\left[V^2(t) - V^2(t_0)\right] - \left[U_2(t) - U_2(t_0)\right] \\ &= \int_{t_0}^{t} \left[\mu\left(\frac{d\eta}{dt}\sin\omega t - \frac{d\xi}{dt}\cos\omega t\right) - \frac{\partial U_2}{\partial t}\right] dt, \end{aligned} \tag{9}$$

where V (t) and V (t_0) denote respectively the velocity of the test particle at time t and at the initial time t_0. $\partial U_2/\partial t$ may be evaluated from equations (7) and (8). When the result is substituted into equation (9), we obtain finally

$$\begin{aligned} &\frac{1}{2}\left[V^2(t) - V^2(t_0)\right] - \left[U_2(t) - U_2(t_0)\right] \\ &= \mu\int_{t_0}^{t} \left[\left(\frac{d\eta}{dt} - \frac{\omega\xi}{r_2^3}\right)\sin\omega t - \left(\frac{d\xi}{dt} + \frac{\omega\eta}{r_2^3}\right)\cos\omega t\right] dt. \end{aligned} \tag{10}$$

It is obvious from equation (10) that the sum of the potential and kinetic energy is no longer constant. Since the integral in equation (10) actually depends upon the path of the test particle, $-U_2$ does not behave exactly like a potential. Thus, it is questionable whether U_2 as given by equation (7) can predict the equilibrium surfaces or the limiting surface of the star whose rotation is non-synchronous to the orbital motion. The reason that the limiting surface has been derived in this way is perhaps due to the fact that the names of potential is often given to $-U_2$. If we avoid to use the name potential and consider only the zero-velocity surfaces, no ambiguity will arise because equation (9) clearly shows that zero-velocity surfaces do not exist in the (ξ, η, ζ) coordinate system.

Both Limber (1963) and Kruszewski (1963) treated the problem rigorously by considering the equations of motion. They have introduced

$$U_3(\xi, \eta, \zeta) = U_2(\xi, \eta, \zeta) + \mu(\eta \sin \omega t - \xi \cos \omega t). \tag{11}$$

In doing so, they can replace the right hand sides of equations (4)-(6) respectively by $\partial U_3/\partial\xi$, $\partial U_3/\partial\eta$, and $\partial U_3/\partial\zeta$. Because of the transformation relation (3), $U_3(\xi, \eta, \zeta)$ can now be expressed in terms of x, y and z. In this new expression for U_3, time appears only implicitly through x,y,z but does not enter explicitly. Consequently, they have independently derived the limiting surface in the (x,y,z) system by considering U_3 as the potential function.

However, we should remember that it is the (ξ, η, ζ) system that is rigidly fixed to the rotating star. Any stationary surface of the non-synchronous rotating star ($1-\mu$ component) must be expressed as a function of ξ , η and ζ alone without the explicit appearance of time. It does not appear that an expression in x, y, and z can represent the stationary surfaces of the non-synchronous component because in the (x,y,z) system the component has a net rotation. Since, as we have seen before, we cannot obtain a potential function in the (ξ, η, ζ) system, we have no means to compute the stationary surfaces of a non-synchronous rotating component star, a conclusion contrary to the previous investigators.

II. Physical Interpretation

Because of the difficulties we have just observed, it is advisable to retain the (x,y,z) coordinate system that rotates with the binary motion.

In this coordinate system we have

for the test particle a simple energy integral

$$2 U_1 - V^2 = C \tag{12}$$

where $U_1(x,y,z)$ is given by equation (2).

We should now recall that the zero-velocity surfaces are labelled by C (e.g., Moulton 1914). Let the C value that is associated with the innermost contact surface S_1 (Kuiper 1941) be C_1. Therefore, all particles inside the S_1 surface with $C < C_1$ (gradually could penetrate the S_1 surface. As a result, these particles will be lost from the star.

Now the $1 - \mu$ component is rotating with an angular velocity ω in the x,y,z coordinate system along the z-axis. Particles in the stellar surface layers have velocities

$$V = (x^2 + y^2)^{1/2} \omega \tag{13}$$

if thermal motion or other kinds of motion are neglected. Hence, all particles in the $1 - \mu$ component star that are above the surface (called the R surface hereafter) given by

$$2 U(x,y,z) - \omega^2 (x^2 + y^2) = C_1 \tag{14}$$

and consequently have C values less than C_1 may be regarded as unstable and could easily escape out of the S_1 surface. Figure 1 illustrates the cross sections in both the xy plane and the xz plane of a few R surfaces for the case $\mu = 0.4$ and ω = 0, 2, 4 and 8. The case $\omega = 0$ is simply the S_1 surface for $\mu = 0.4$. The elongated shape of the cross sections in the xz plane is easy to understand because those particles near the equatorial plane that have the highest linear velocities due to non-synchronous rotation are the easiest to be ejected out of the S_1 surface.

However, it should be noted that the R surfaces represent neither the limiting nor equilibrium surface. Particles below the R surface for a given value of ω can move up although they cannot escape through the S_1 surface without collisions. Statistically, we can state that most particles that are originally located above the R surface are lost and are replaced by particles coming from below the R surface. These new particles will naturally fill up the entire lobe of the S_1 surface. Therefore, the limiting surface of the $1 - \mu$ component is still given by the S_1 surface. Since the angular momentum per unit mass due to the axial rotation of the $1 - \mu$ component decreases downward in the star if it rotates as a rigid body, the new material above the R surface will rotate less rapidly (with respect to the xyz system) than the old material before the latter's escape. The simultaneous transfer of mass from below the R surface to above the R surface and from inside the S_1 surface to outside the S_1 surface will continue until the orbital revolution and the axial rotation become synchronized. Perhaps this is one of the most effective mechanisms for synchronizing orbital revolution and axial rotation of close binary stars. Therefore, non-synchronization observed in those close binaries must be a temporary phenomenon triggered by rapid evolution of the component star itself as is suggested by Kopal (1959). But even at the time the two kinds of motion are temproarily out of step, the shape of the component is still given by the S_1 surface according to the present analysis.

Finally, one may question our argument on the ground that it has not proven that the particles are necessarily flying away from the star. If all particles on some surface of a non-synchronously rotating star should tend to move inward, this surface could be regarded as a stable surface for the star. This is however not true because when the particles move inward they collide with other particles. Since the average C value (the C value of each particle being defined by Eq. [12]) of all particles participating in a collision remains constant (Huang 1965) and since directions of the velocities will be modified after the collision, the chance of escape from the surface increases with time whatever are the directions of their initial velocities (in the xyz system). Therefore, our conclusion about the non-synchronously rotating star is valid without qualification for the Roche model.

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Figure 1 - The R surfaces for different values of ω (0, 2, 4, 8) for $\mu = 0.4$. The R surfaces are defined by equation (14). The case $\omega = o$ represents simply the innermost contact surface (denoted by S_1). When the rotation of a component star is not synchronous with its orbital motion, the gaseous particles above the R surface may escape the S_1 surface and are replaced by gaseous particles moving out from below the R surface. This process will go on until the star finally becomes synchronized to its orbital motion.